BIT5204
Computer Modelling of Continuous Physical Systems

Simulation of Passive and Active Suspension Systems

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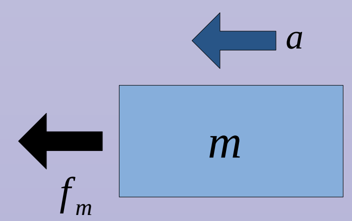
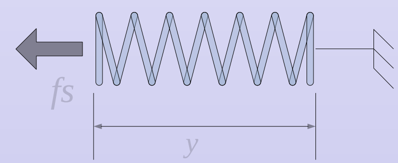
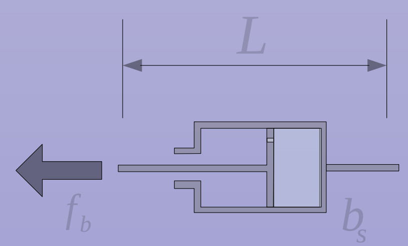
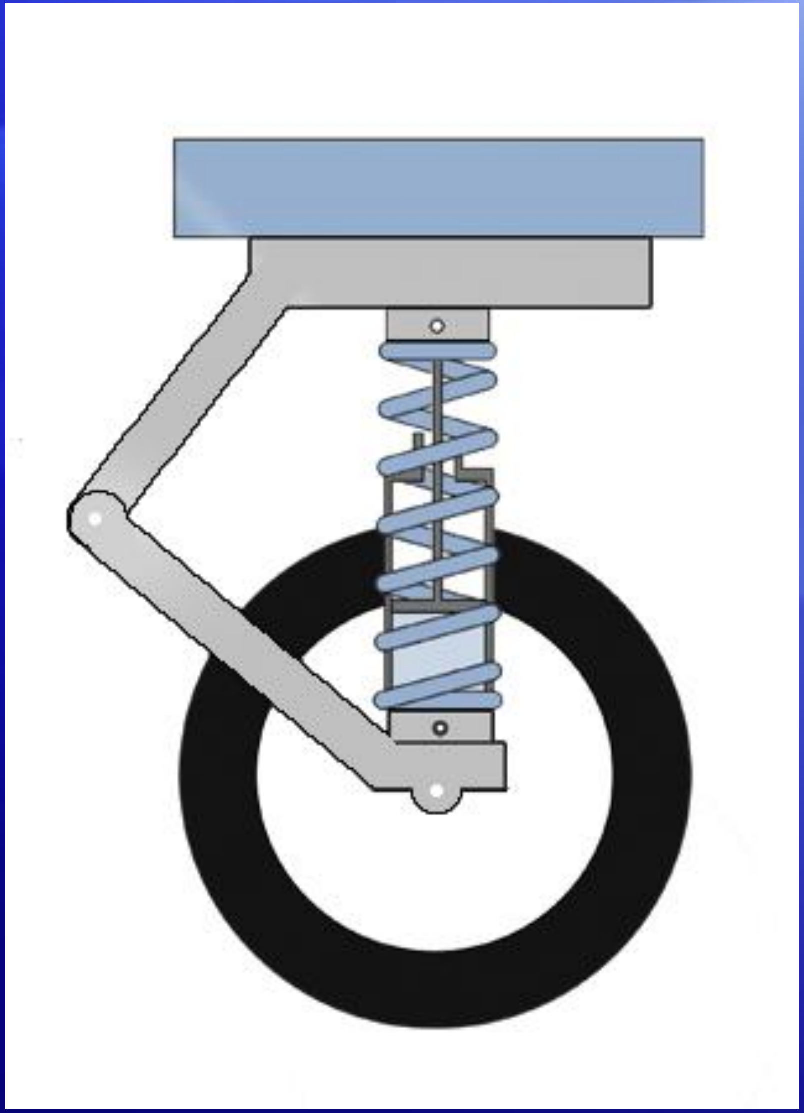
Agenda

- Basic Concepts
- The Suspension System Equations
- Solution of ODE
- Control System Simulator Program
- Simulation of Passive Suspension System
- Control of a Suspension System – Active Suspension Systems
- Active Suspension System Equations
- Determining a Ride Comfort Measure
- Negative Feedback Controller
- The PID Controller
- PID Controlled Suspension System
- System and Simulation Limitations

Basic concepts

Element	Description	Diagram	Equation
Mass	**Newton's second Law** The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object		$f_m = m\frac{d^2x}{dt^2}$
Spring	**Hooke's Law:** The restoring force due to a spring is proportional to the length that the spring is stretched, and acts in the opposite direction		$f_s = ky$
Damper	The force required to move the damper is proportional to rate of change of its length		$f_b = b\frac{dL}{dt}$

Physical system



Purpose of suspension Systems:

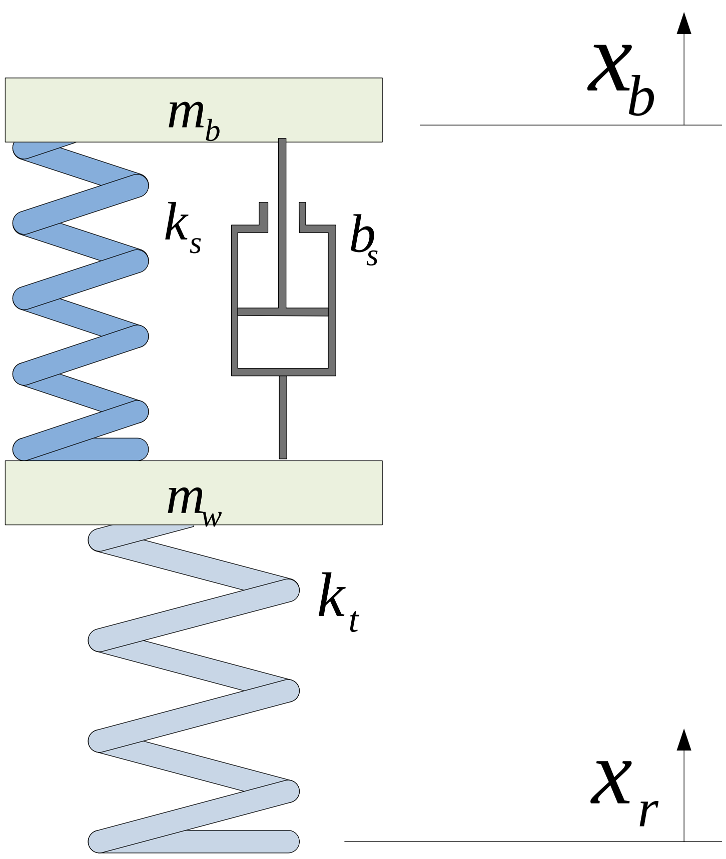
- Comfortable ride – reduces effect of road bumps.
- Control – keeps wheels on ground

Dependent and Independent suspension types

Independent Suspensions

- MacPherson Strut
- Double Wishbone Suspension Systems
- …others

Quarter Car Suspension Model



m_b represents the car chassis (quarter of car body mass)

m_w represents the mass of a wheel

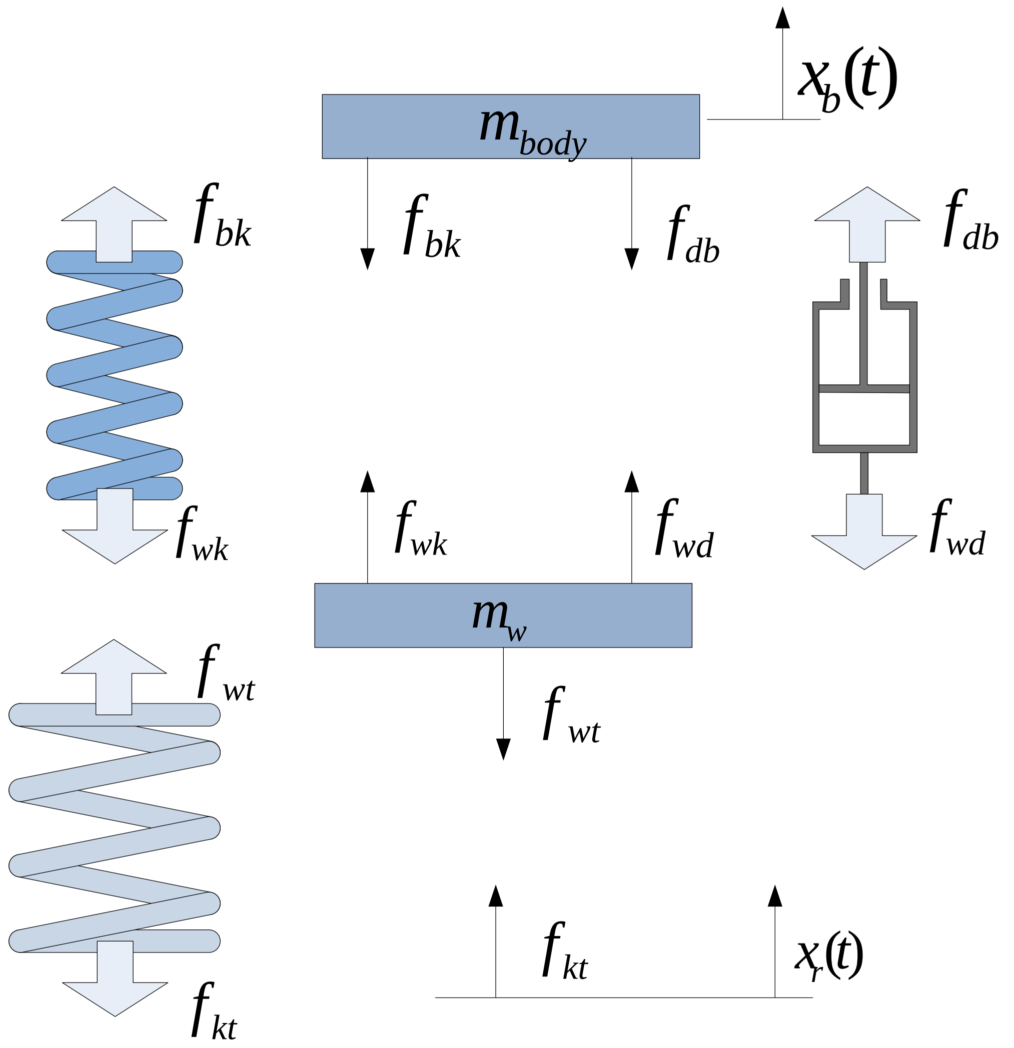
The two masses are connected together by a suspension that is made up of a spring of stiffness k_s and absorber having a damping factor b_s

k_t models the compressibility of the tyre.

x_b is the car body travel.

x_r is the road disturbance.

Examination of Physical Model



$$f_{bk} = f_{body \to spring} = f_{spring \to body}$$
$$f_{bd} = f_{body \to damper} = f_{damper \to body}$$

$$f_{wk} = f_{wheel \to spring} = f_{spring \to wheel}$$
$$f_{wd} = f_{wheel \to damper} = f_{damper \to wheel}$$

$$f_{wt} = f_{wheel \to tyre} = f_{tyre \to wheel}$$

Examination of Physical Model

Using Newton's second Law, or $\sum f = ma$ around m_b

$$m_b \ddot{x}_b = -k_s(x_b - x_w) - b_s(\dot{x}_b - \dot{x}_w)$$

Hence

$$\ddot{x}_b = -\frac{k_s}{m_b}(x_b - x_w) - \frac{b_s}{m_b}(\dot{x}_b - \dot{x}_w)$$

And around m_w

$$m_w \ddot{x}_w = k_s(x_b - x_w) - b_s(\dot{x}_b - \dot{x}_w) - k_t(x_w - x_r)$$

Therefore

$$\ddot{x}_w = \frac{k_s}{m_w}(x_b - x_w) + \frac{b_s}{m_w}(\dot{x}_b - \dot{x}_w) - \frac{k_t}{m_w}(x_w - x_r)$$

Examination of Physical Model

Assume

$$x_1 = x_b$$
$$x_2 = \dot{x}_1 = \dot{x}_b$$
$$x_3 = x_w$$
$$x_4 = \dot{x}_3 = \dot{x}_w$$

$$\dot{x}_2 = -\frac{k_s}{m_b}(x_1 - x_3) - \frac{b_s}{m_b}(\dot{x}_1 - \dot{x}_3)$$

$$\boxed{\therefore \ \dot{x}_2 = -\frac{k_s}{m_b}(x_1 - x_3) - \frac{b_s}{m_b}(x_2 - x_4)}$$

$$\dot{x}_4 = \frac{k_s}{m_w}(x_1 - x_3) + \frac{b_s}{m_w}(\dot{x}_1 - \dot{x}_3) - \frac{k_t}{m_w}(x_3 - x_r)$$

$$\boxed{\therefore \ \dot{x}_4 = \frac{k_s}{m_w}(x_1 - x_3) + \frac{b_s}{m_w}(x_2 - x_4) - \frac{k_t}{m_w}(x_3 - x_r)}$$

Examination of Physical Model

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{k_s}{m_b}x_1 - \frac{b_s}{m_b}x_2 + \frac{k_s}{m_b}x_3 + \frac{b_s}{m_b}x_4$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{k_s}{m_w}x_1 + \frac{b_s}{m_w}x_2 - \left(\frac{k_t}{m_w} + \frac{k_s}{m_w}\right)x_3 - \frac{b_s}{m_w}x_4 + \frac{k_t}{m_w}x_r$$

Examination of Physical Model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_b} & -\frac{b_s}{m_b} & \frac{k_s}{m_b} & \frac{b_s}{m_b} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_w} & \frac{b_s}{m_w} & -\frac{k_t}{m_w} - \frac{k_s}{m_w} & -\frac{b_s}{m_w} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_t}{m_w} \end{bmatrix} x_r$$

$$\underline{\dot{x}} = \underline{A}\,\underline{x} + \underline{B}\,x_r$$

$$x_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\underline{y} = \underline{C}\,\underline{x} + 0x_r$$

Solution of ODE - Euler

Consider a given ODE $\dot{y} = f(x, y)$

Where $y(0) = y_0$

And
$$x_1 = x_0 + h$$
$$x_2 = x_0 + 2h$$

Consider a Taylor series expansion in the neighbourhood of x(0)

$$y(x+h) = y(x) + h\dot{y}(x) + \frac{h^2}{2!}\ddot{y}(x) + \cdots$$

If h is very small,
$$y(x+h) \approx y(x) + h\dot{y}(x)$$
$$= y(x) + hf(x, y)$$

Therefore
$$y_1 = y_0 + hf(x_0, y_0)$$
$$y_2 = y_1 + hf(x_1, y_1)$$

$$\boxed{y_{n+1} = y_n + hf(x_n, y_n)}$$

Solution of ODE – Runge-Kutta

Euler’s method is derived from a truncated Taylor series, therefore

- Each step has a local discretization error, dependent of the chosen h and the value of x at that point.
- This error is accumulated when Euler method is applied for several steps of size h.

- Better solution Runge-Kutta

Solution of ODE – Runge-Kutta

Slope is computed at four places within each step

$$k_1 = f'(x_j, y_j)$$

$$k_2 = f'(x_j + \frac{h}{2}, y_j + k_1 \frac{h}{2})$$

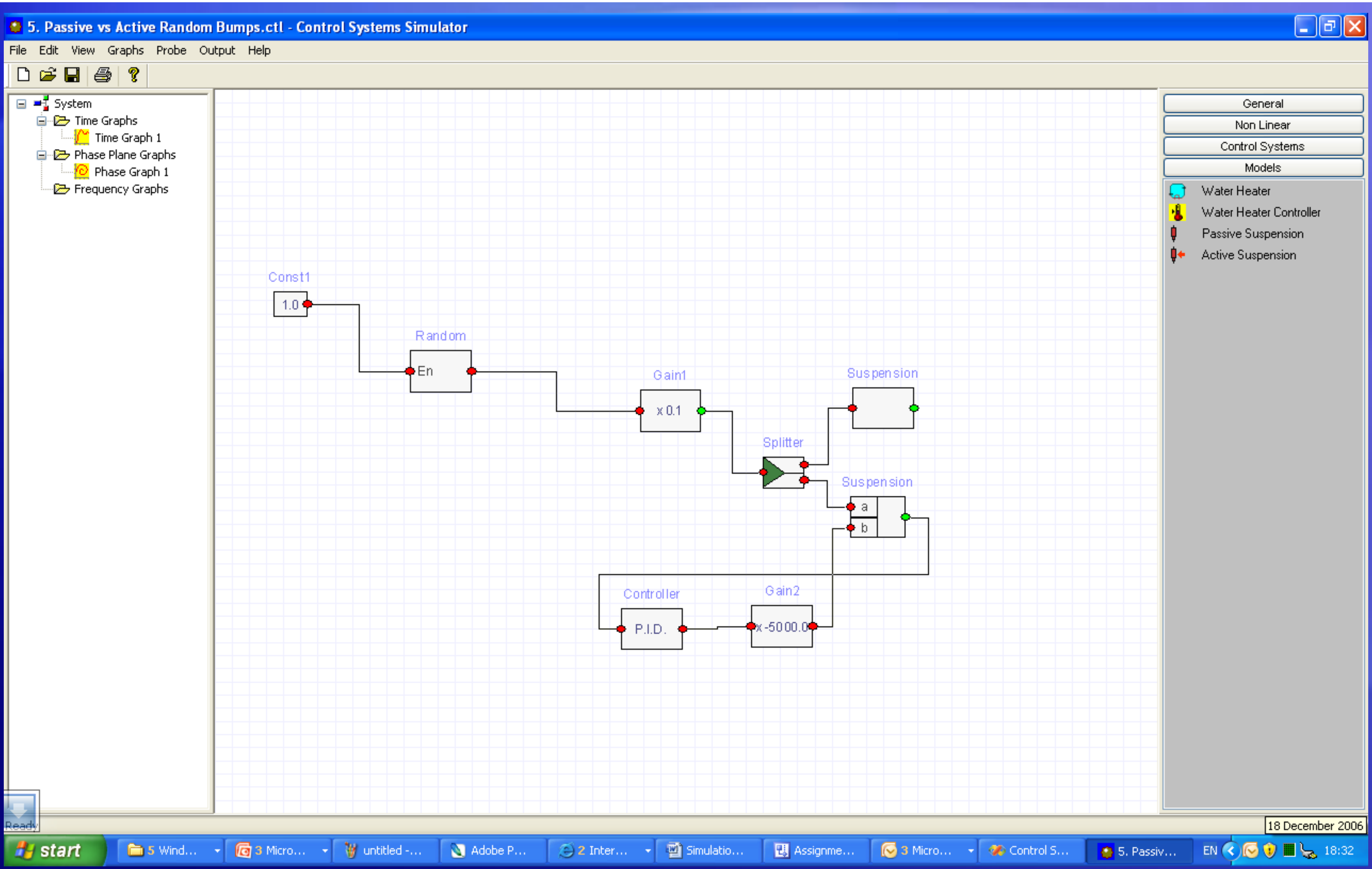
$$k_3 = f'(x_j + \frac{h}{2}, y_j + k_2 \frac{h}{2})$$

$$k_4 = (x_j + \frac{h}{2} y_j + k_3 h)$$

Use weighted averages of slopes to obtain next value

$$\boxed{y_{j+1} = y_j + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)}$$

Control Systems Simulator Program



Simulation of Passive and Active Suspension Systems

CSS Operation

- **Three Sections**
  - System hierarchy view
  - User area
  - System toolbox.

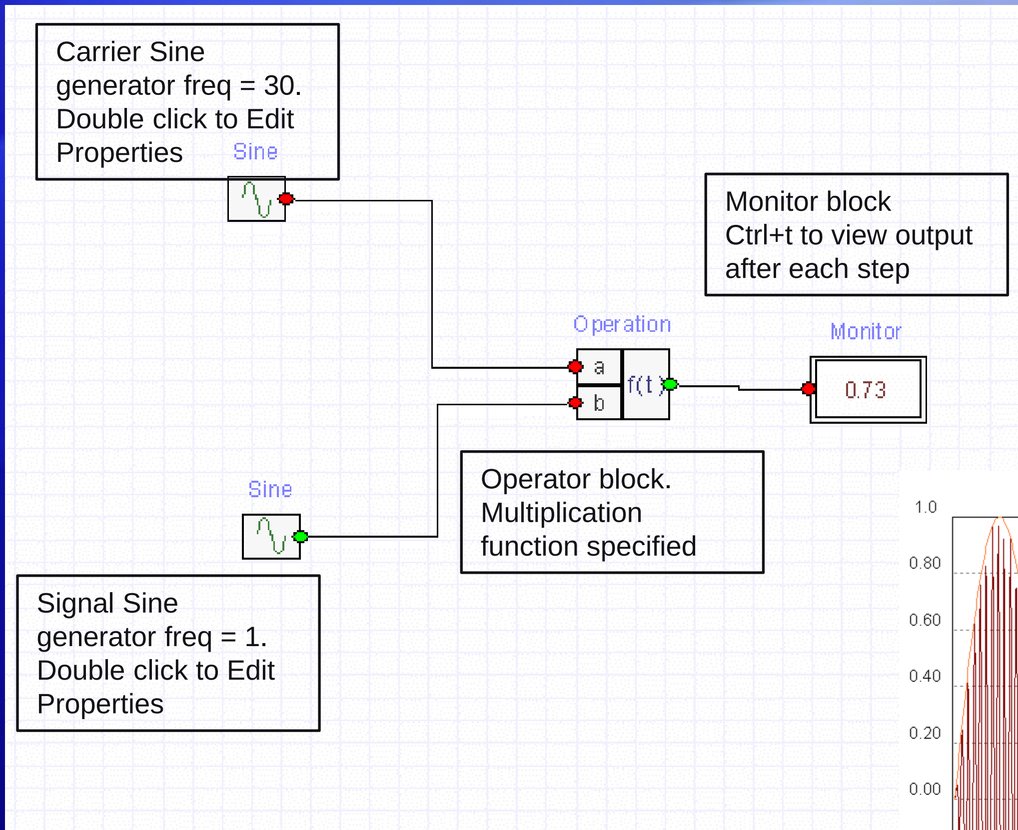
- **Modes of operation**
  - **Stepping mode**. Graphs are not utilized, instead monitor blocks are used to view a node's output.
  - **Execution mode**. Graphs are employed to visualize the system's response. The simulation is executed from time=0 to the specified final time using the specified time step

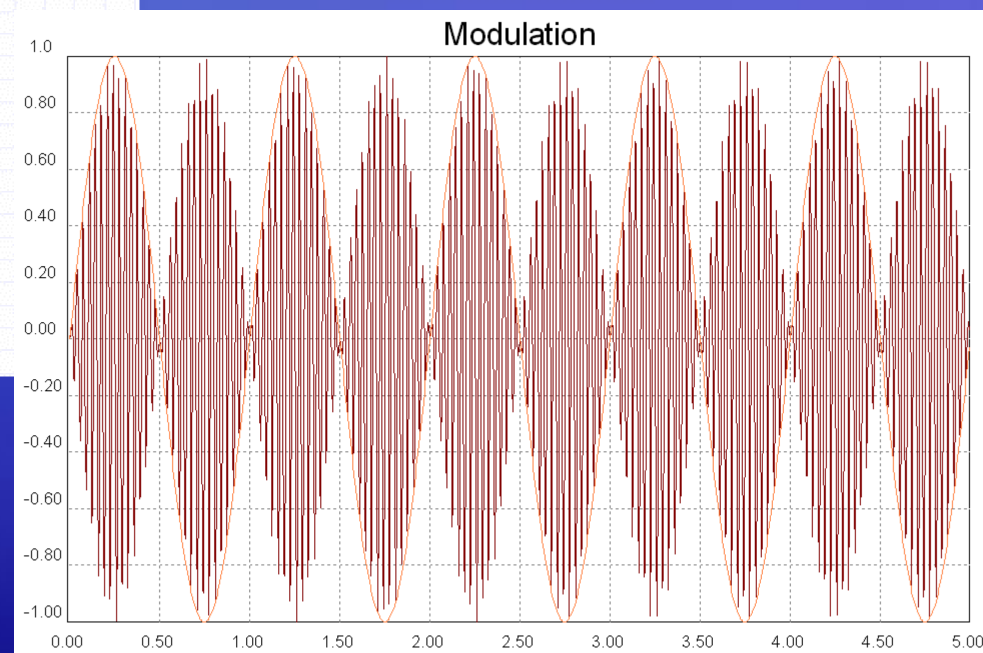
Technologies, IDE

- C++, MFC, Visual Studio 2005

- **Demo 1**

Demo1





Press Ctrl+G to execute Simulation and switch to graph view (double click on graph in system hierarchy view)

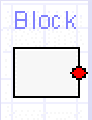
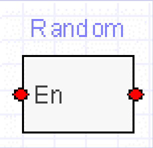
CSS Entities

- Node
  - Can be Input or Output.
  - Holds a value that can be used for calculation (if input node) or can be passed to other nodes (if output node).
- Control Block
  - Container of Input Nodes and Output Nodes.
  - Calculation modifies Output Nodes’ values based on Input Nodes’ Values and time.
- Connection
  - Connects an Input Node to an Output Node.
  - Output Nodes values are fed to connected input node every time step.

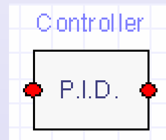
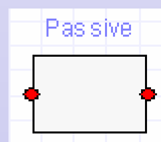
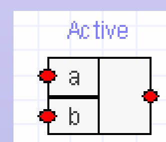
CSS Entities

- Graph Trace
  - Collection of points from a Node that are drawn on a graph.
  - Values are refreshed with every simulation run.
- Graph
  - Collection of Graph Traces.
  - Defines bounds.
  - Currently two types; Time Graph and Phase Plane Graph
- Probe
  - Connects a Block Node to a Graph
  - Trace is Created with every Probe
  - Possible to have many-to-many relationship between nodes and Graphs (Node output can be depicted on many graphs and a graph can contain many node outputs)

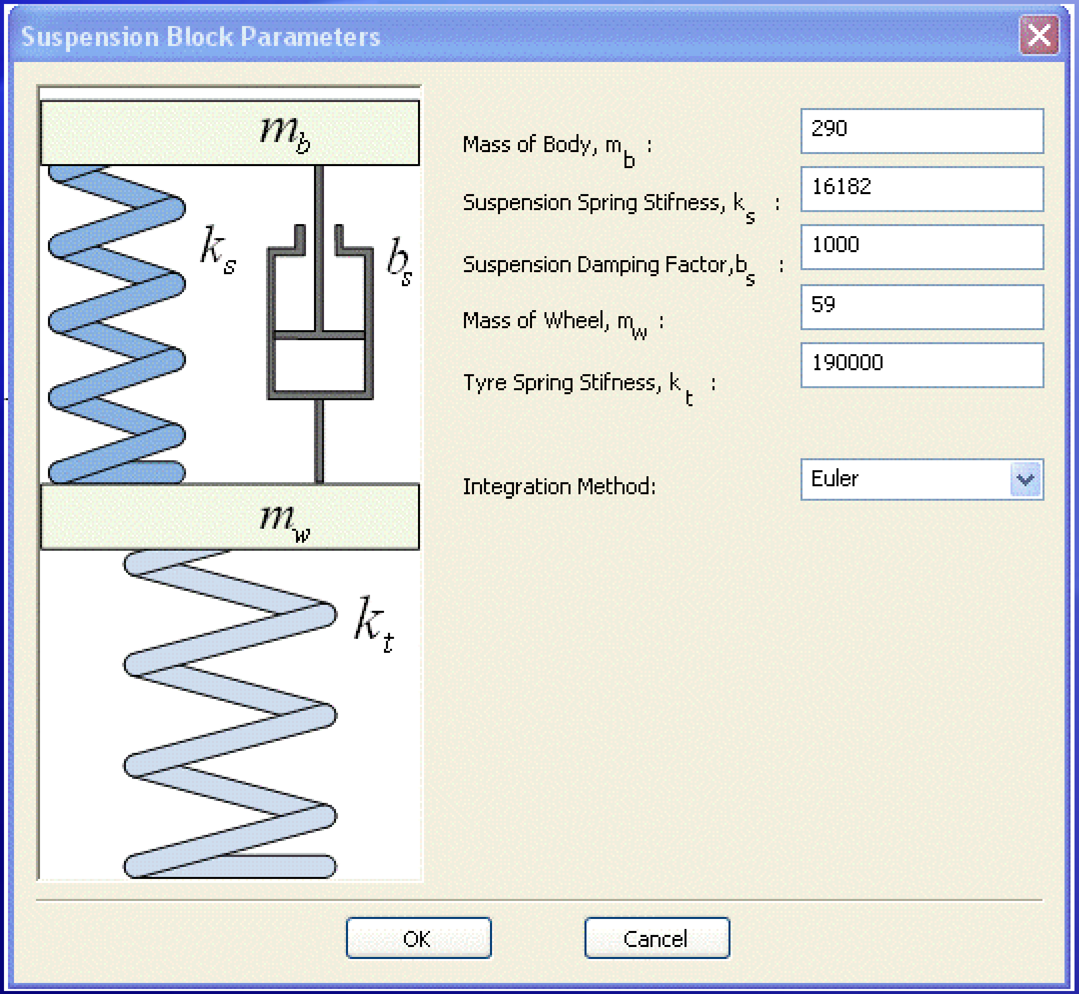
CSS Blocks

Library	Block		Block Description
General Library	Read Function from file		Reads an input function from a file. Function is a system of piecewise linear functions. Parameters: File name File Format; StartTime[:]EndTime[:]Function[new line] Example: 0:1:0 1:1.5:t-1 1.5:2:0.5 2:2.5:2.5-t
	Random		Output a Random function Parameters: Service Time Minimum Value Service Time Maximum Value Inter Arrival Time Minimum Value Inter Arrival Time Maximum Value File name to store random function in a file Input: Enable; must be greater than zero for random function to execute.

CSS Blocks

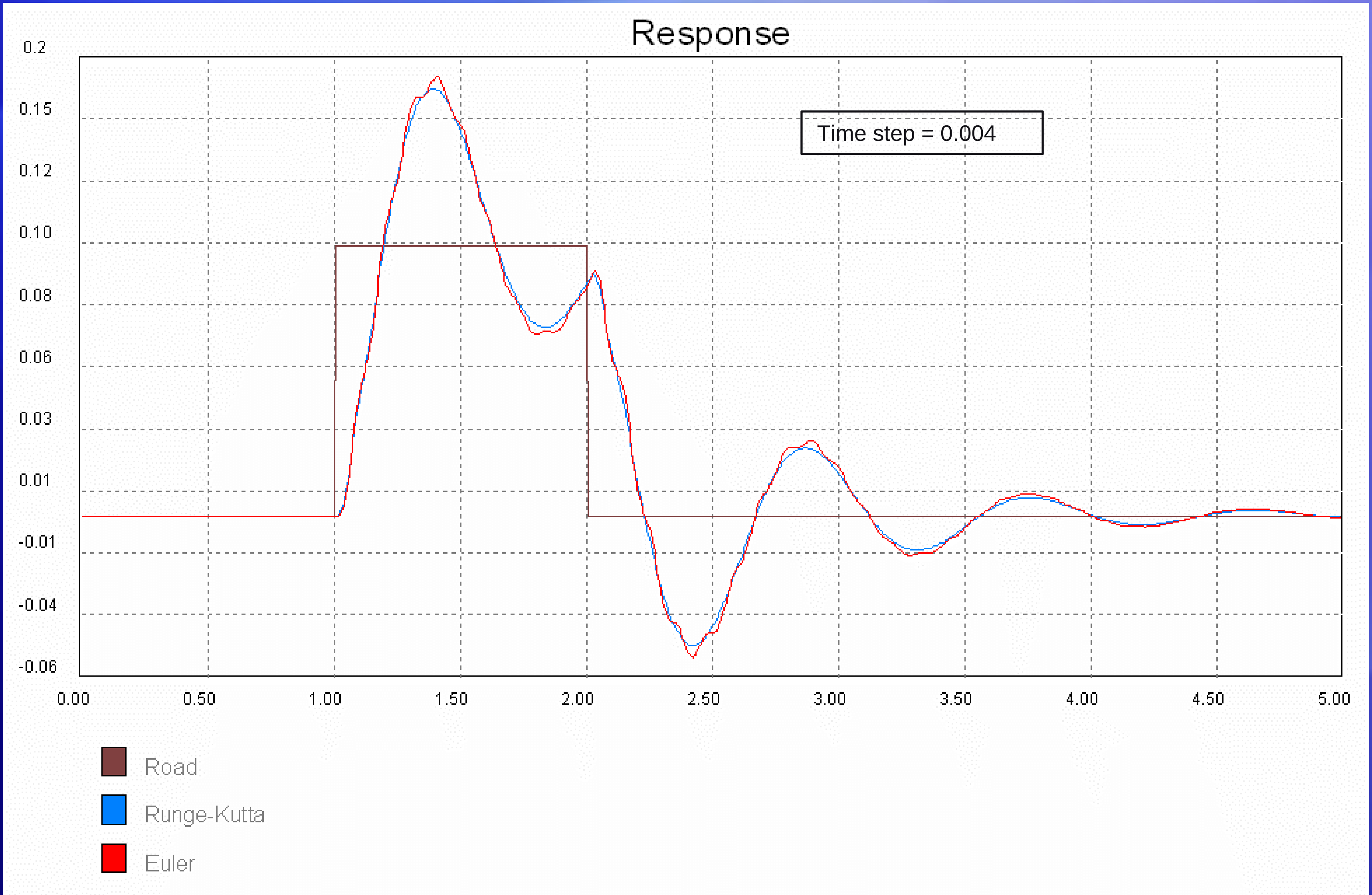
Library	Block		Block Description
Control System Library	PID Controller		Proportional Integral Derivative controller. I Input parameters are P, D & I constants
Models Library	Passive Suspension System		Passive Suspension System. Input: Road Profile Output: Chassis displacement Parameters: Mass of Body Suspension Spring Stiffness Suspension Damping Factor Mass of Wheel Tyre Spring Stiffness Integration Method (Euler or Runge-Kutta)
	Active Suspension System		Active suspension system Input: Road Profile Control Input Output: Chassis displacement

Passive Suspension System



- Passive System is represented by one single input – single output block
- Input is road profile
- Output is chassis displacement
- Selection of spring stiffness values, damping factor , etc. [Lin97]
- Demo 2, 3 & 4

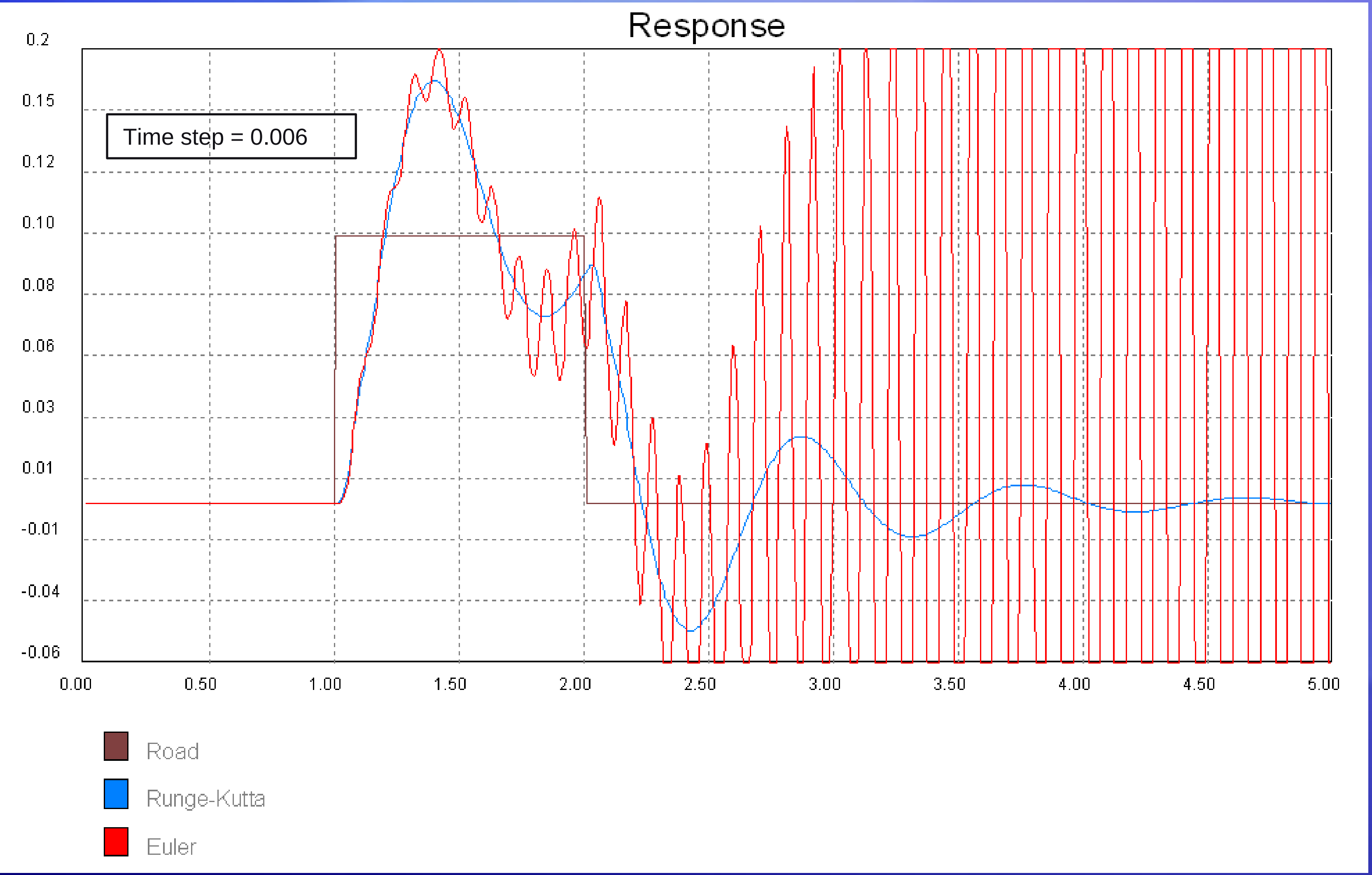
Response of PSS – Euler vs Runge-Kutta



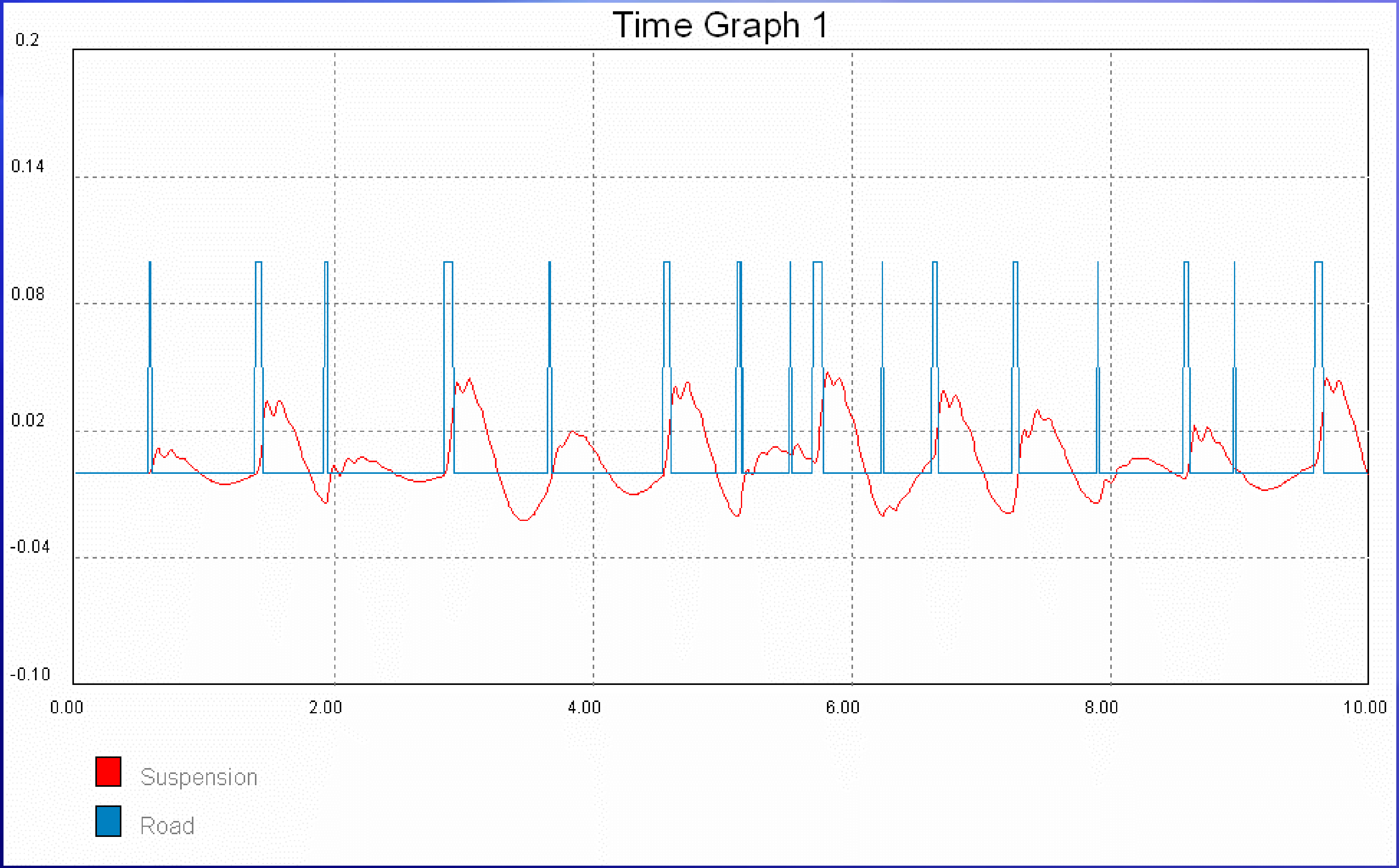
Response of PSS – Euler vs Runge-Kutta

Simulation of Passive and Active Suspension Systems

Response of PSS – Euler vs Runge-Kutta

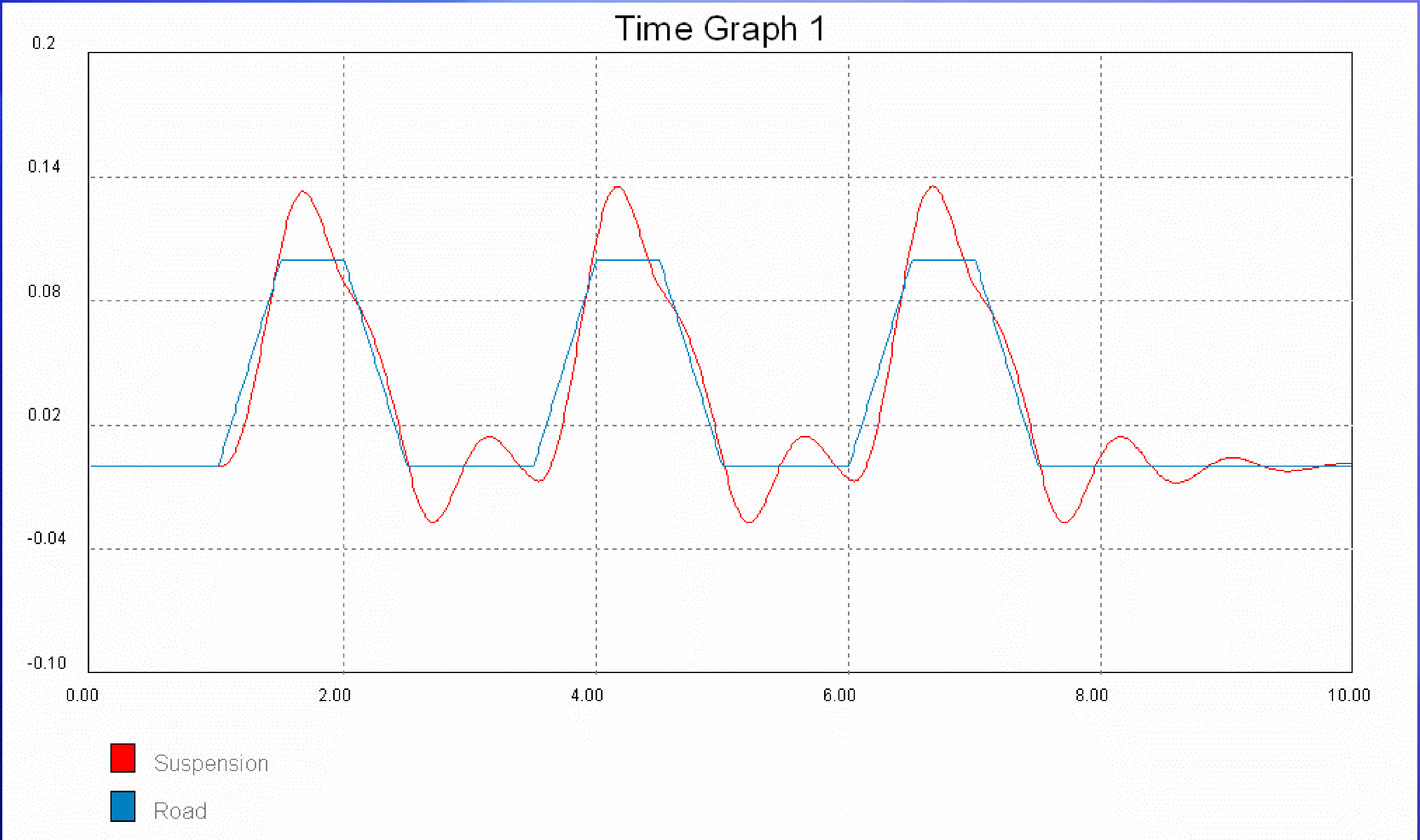


Response of PSS



Simulation of Passive and Active Suspension Systems

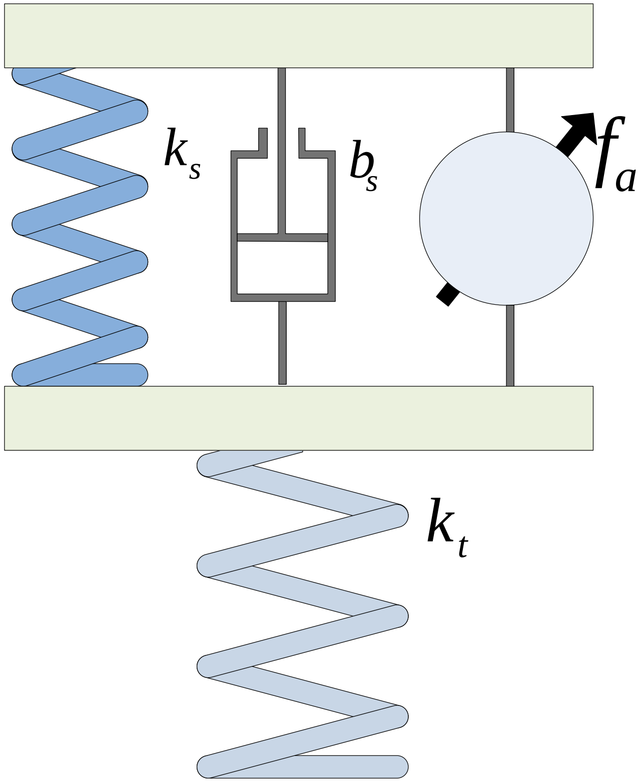
Response of PSS



Passive Suspension Limitations

- Characteristics of passive suspension systems are fixed by manufacturer
- Depend on the environment where suspension is deployed
- Passive suspension design is a compromise between comfortable ride and vehicle handling
- Heavy damping yields good vehicle stability but most of the road bumps are transferred to the body. Hence ride is not enjoyable and cargo might be damaged.
- Lightly damped suspension will improve comfort but reduces stability especially in turns.
- However it is easy to manufacture and relatively cheap

Active Suspension System



An actuator is added so that passenger comfort and vehicle stability are improved

Design is more complex

Increased cost to manufacture

Energy is required to drive source. Active suspension systems have large energy demand.

Other configurations exist (energy source alone or energy source + spring)

Advantage of configuration examined is that energy source can be switched off and suspension action is a passive one.

Active Suspension Analysis

Modified equations:

$$m_b \ddot{x}_b = f_a - k_s(x_b - x_w) - b_s(\dot{x}_b - \dot{x}_w)$$

$$m_w \ddot{x}_w = -f_a + k_s(x_b - x_w) + b_s(\dot{x}_b - \dot{x}_w) - k_t(x_w - x_r)$$

Therefore,

$$\dot{x}_2 = -\frac{k_s}{m_b}x_1 - \frac{b_s}{m_b}x_2 + \frac{k_s}{m_b}x_3 + \frac{b_s}{m_b}x_4 + \frac{f_a}{m_b}$$

$$\dot{x}_4 = \frac{k_s}{m_w}x_1 + \frac{b_s}{m_w}x_2 - \left(\frac{k_t}{m_w} + \frac{k_s}{m_w}\right)x_3 - \frac{b_s}{m_w}x_4 + \frac{k_t}{m_w}x_r + \frac{f_a}{m_w}$$

Active Suspension Analysis

Modified s.s. equations:

$$\underline{\dot{x}} = \underline{A}\,\underline{x} + \underline{B}x_r + \underline{F}f_a$$

$$\underline{y} = \underline{C}\,\underline{x} + 0x_r$$

Where

$$F = \begin{bmatrix} 0 \\ \frac{1}{m_a} \\ 0 \\ \frac{1}{m_w} \end{bmatrix}$$

Determining Ride Comfort

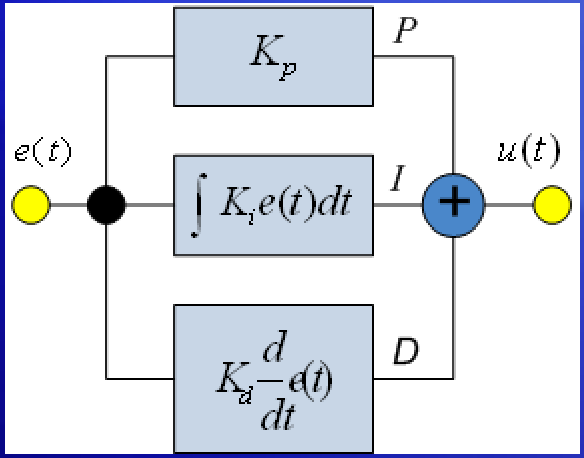
Factors indicating suspension performance resulting in a comfortable drive:

1. Maximum chassis overshoot resulting from a road bump.
2. Rate of change of chassis displacement as an effect of road bumps.
3. Frequency response of chassis displacement.

- Variation of chassis displacement and rate of change of displacement as they vary with time can be viewed in a phase-plane plot.
- A single bump having a fixed length is used to observe the response of suspension systems.
- The phase plane plot resulting from this disturbance is closed (suspension will eventually settle down in its initial position).
- The area enclosed in the plot will indicate the displacement vs. rate of change of displacement cyclic integral. This value will be used as a ride comfort index.

$$\lim_{n\to\infty}\sum_{i=1}^{n}|f(x_{i+1})|(x_{i+1}-x_i)$$

Controlling the Active Suspension



Negative feedback

- Input the an amplified chassis displacement in anti phase.

PID Controller

- Combination of PI and PD controllers; hence it is a lag-lead compensator [Ogata]
- PID used in systems that required improvements in both transient and steady state performances
- PID Tuning

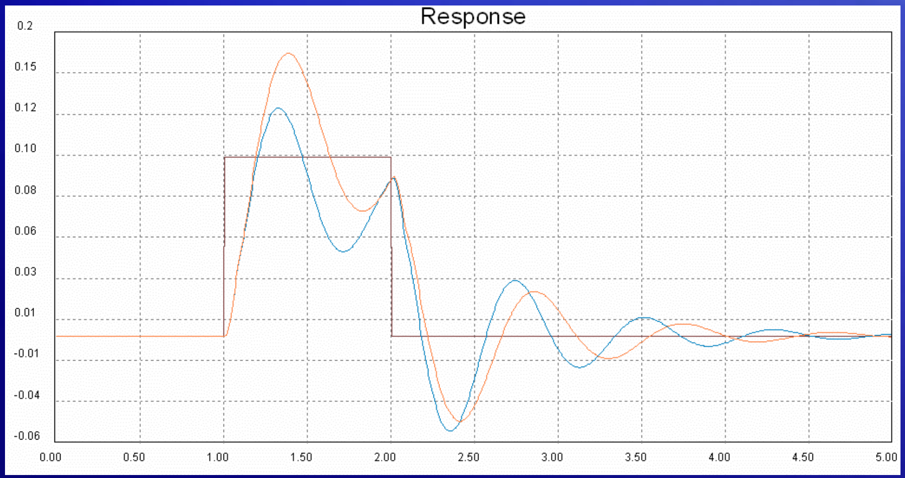
Other Possible Systems

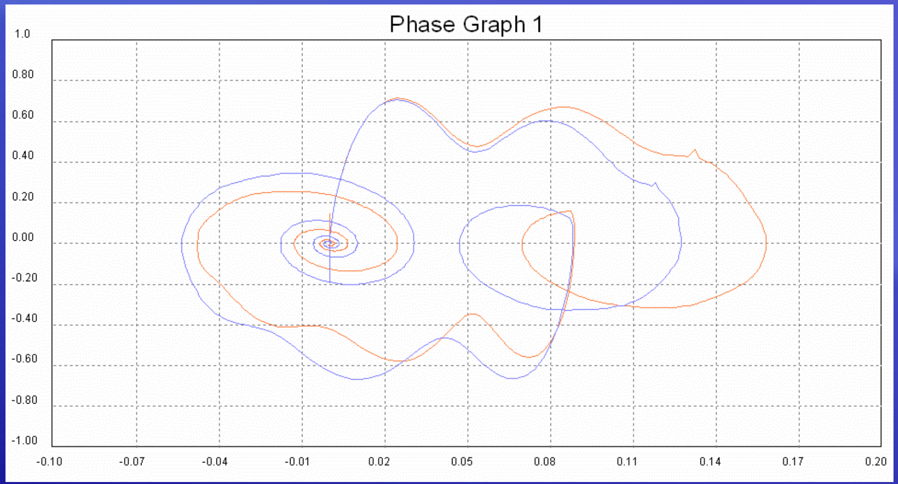
Comparison: Neg. Feedback Control vs. Passive

Demo 7

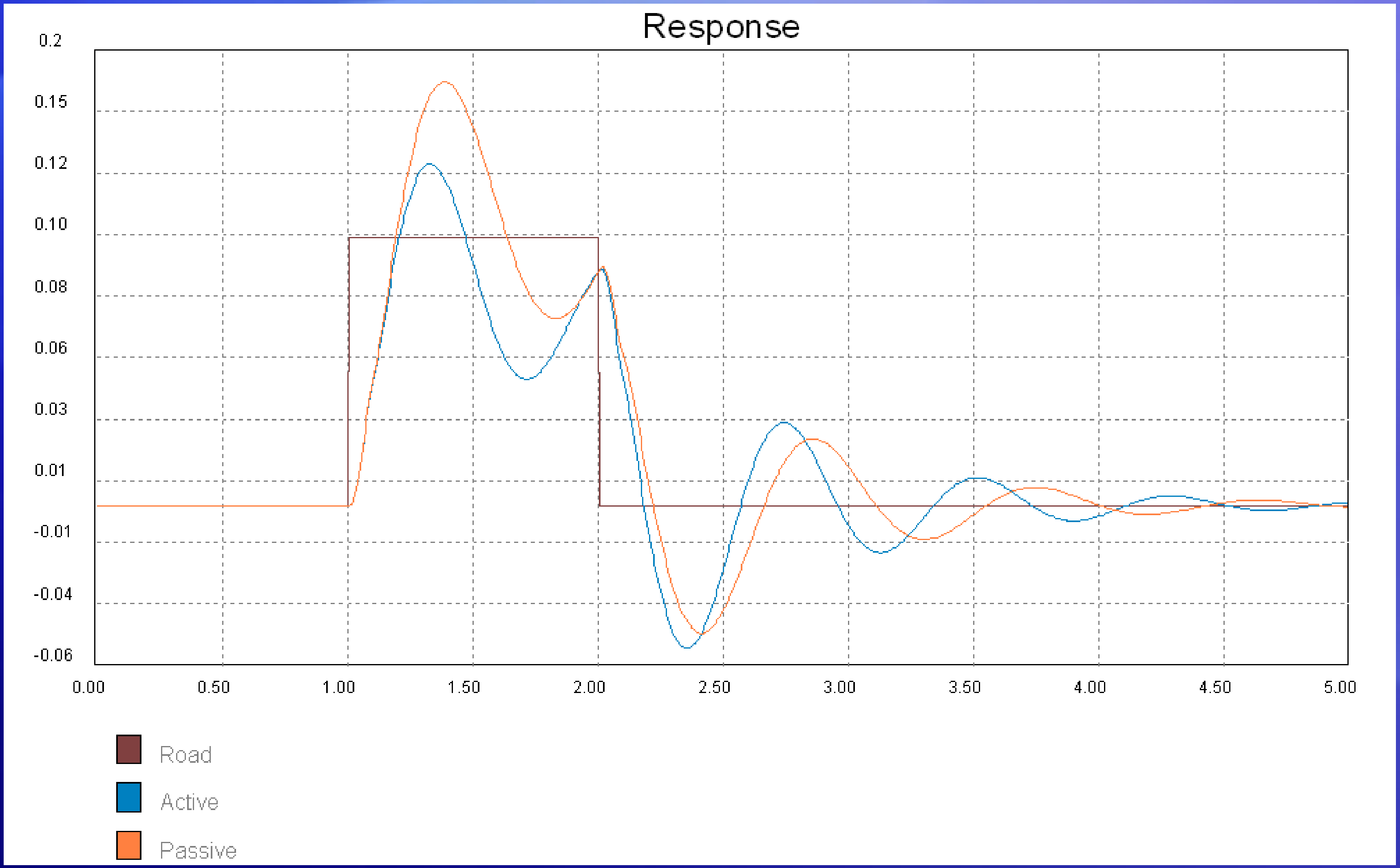
1. Passive overshoot higher than active.
2. Active undershoot higher than passive.
3. Active response has higher differential values.

4. Practically no difference in ride comfort index.



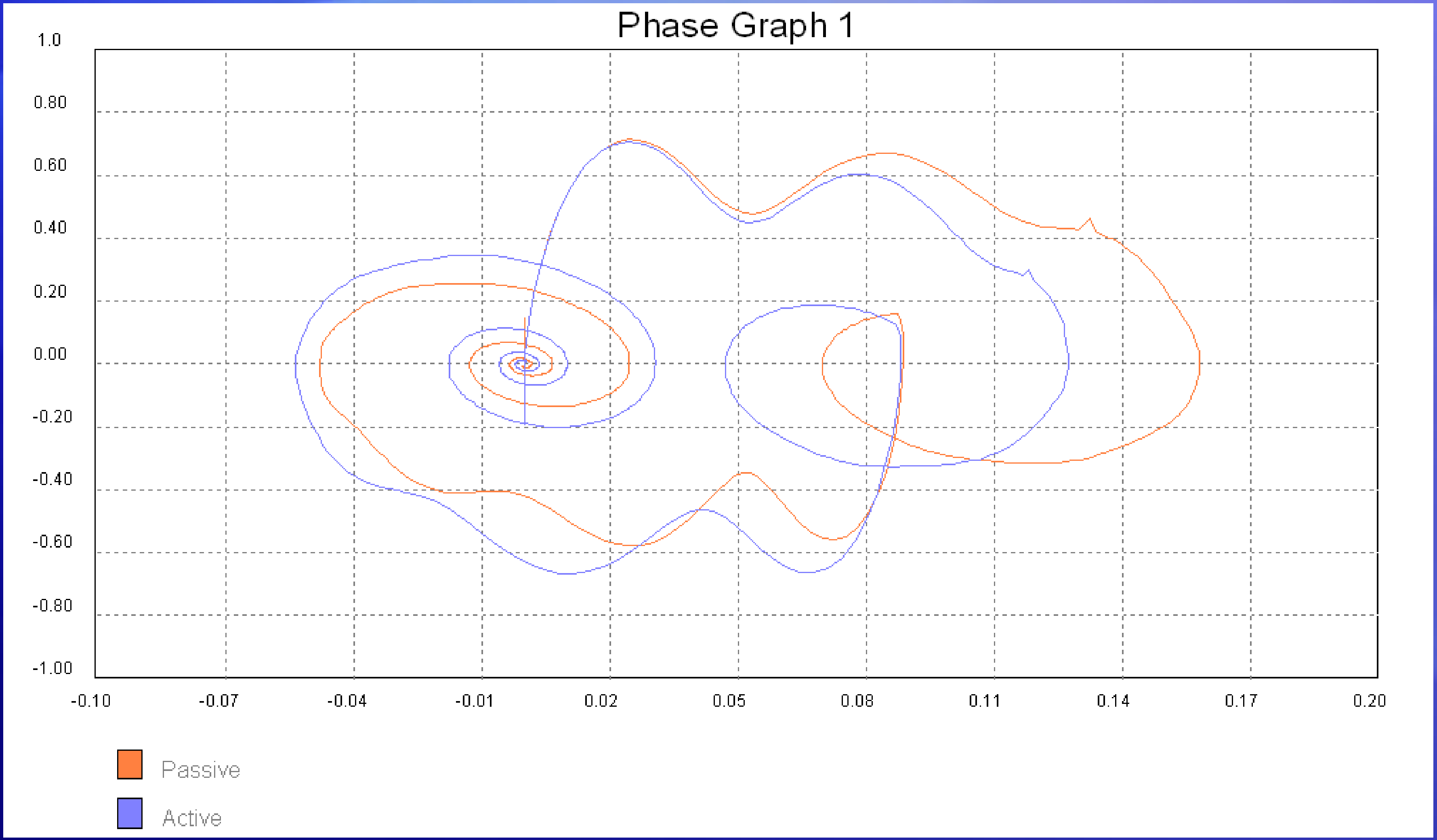


Comparison: Neg. Feedback Control vs. Passive



Simulation of Passive and Active Suspension Systems

Comparison: Neg. Feedback Control vs. Passive



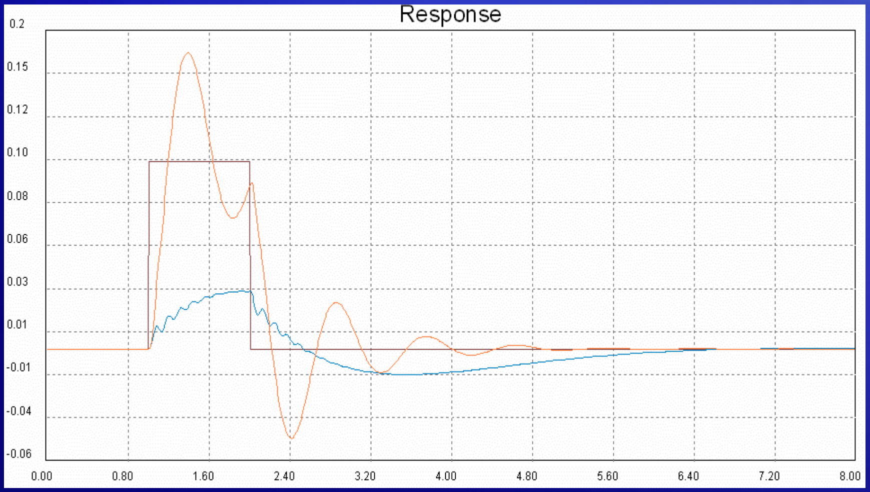
Simulation of Passive and Active Suspension Systems

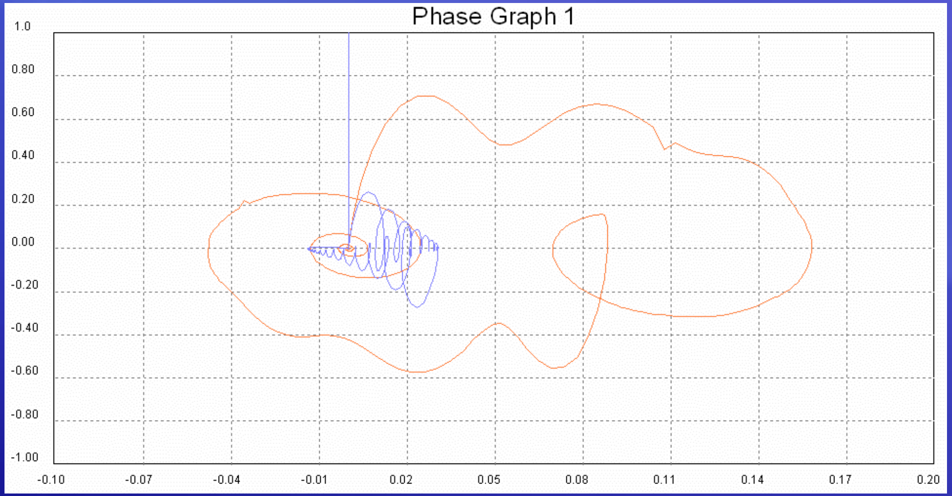
Comparison: PID Control vs. Passive

Demo 8

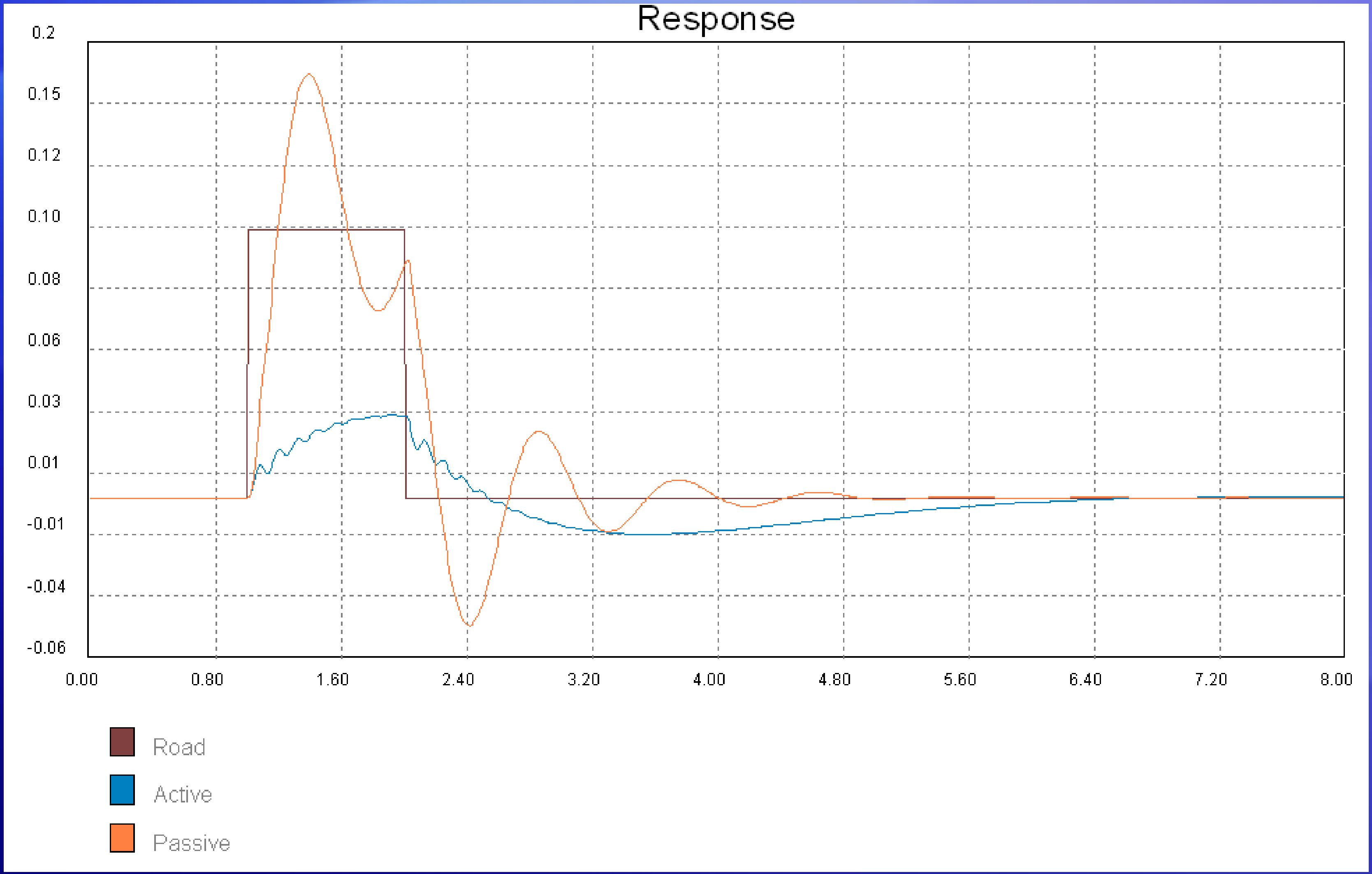
1. Large improvement in overshoot and undershoot values.
2. High Frequency components introduced.

3. Ride Comfort improvement by an order of magnitude.



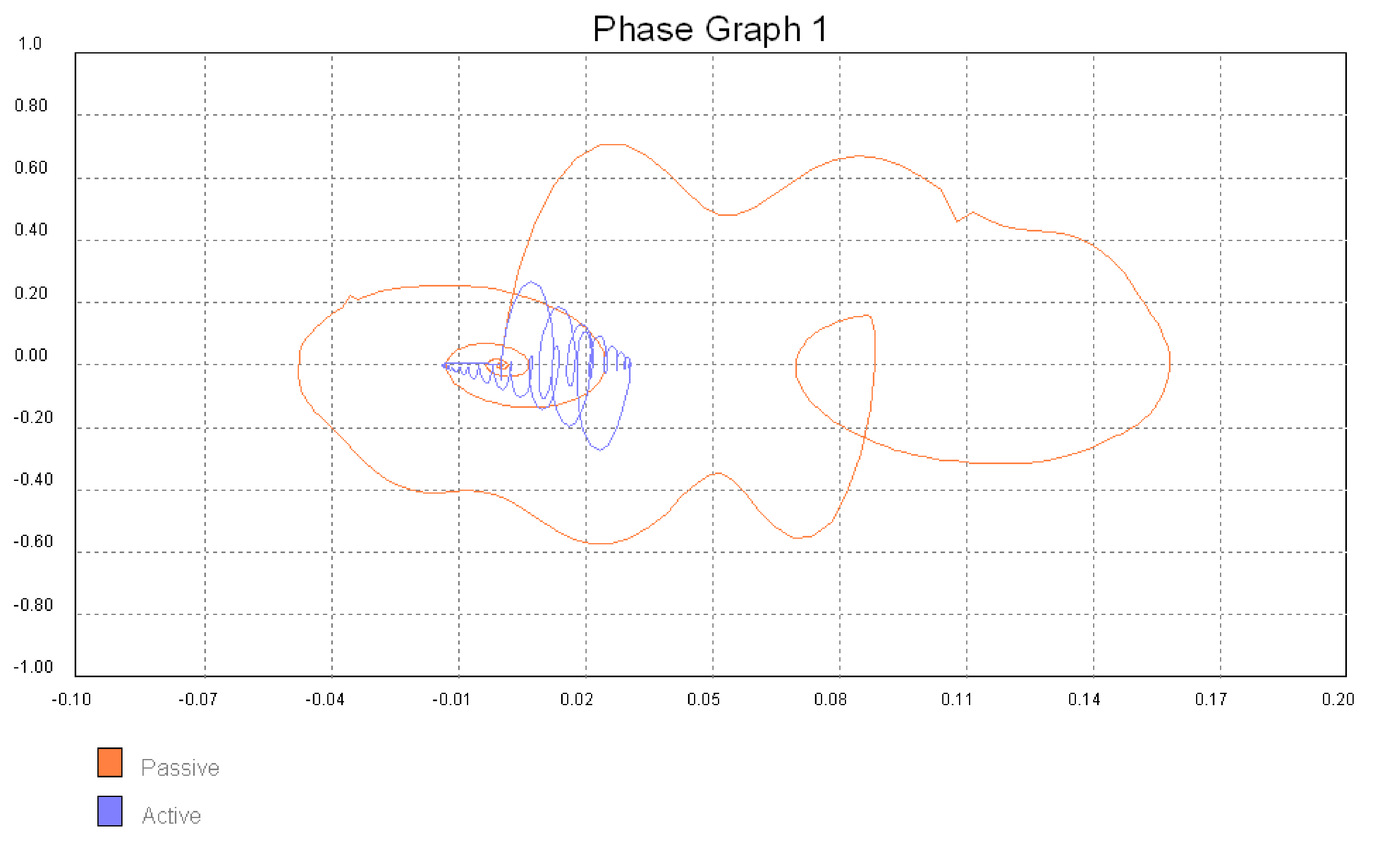


Comparison: PID Control vs. Passive



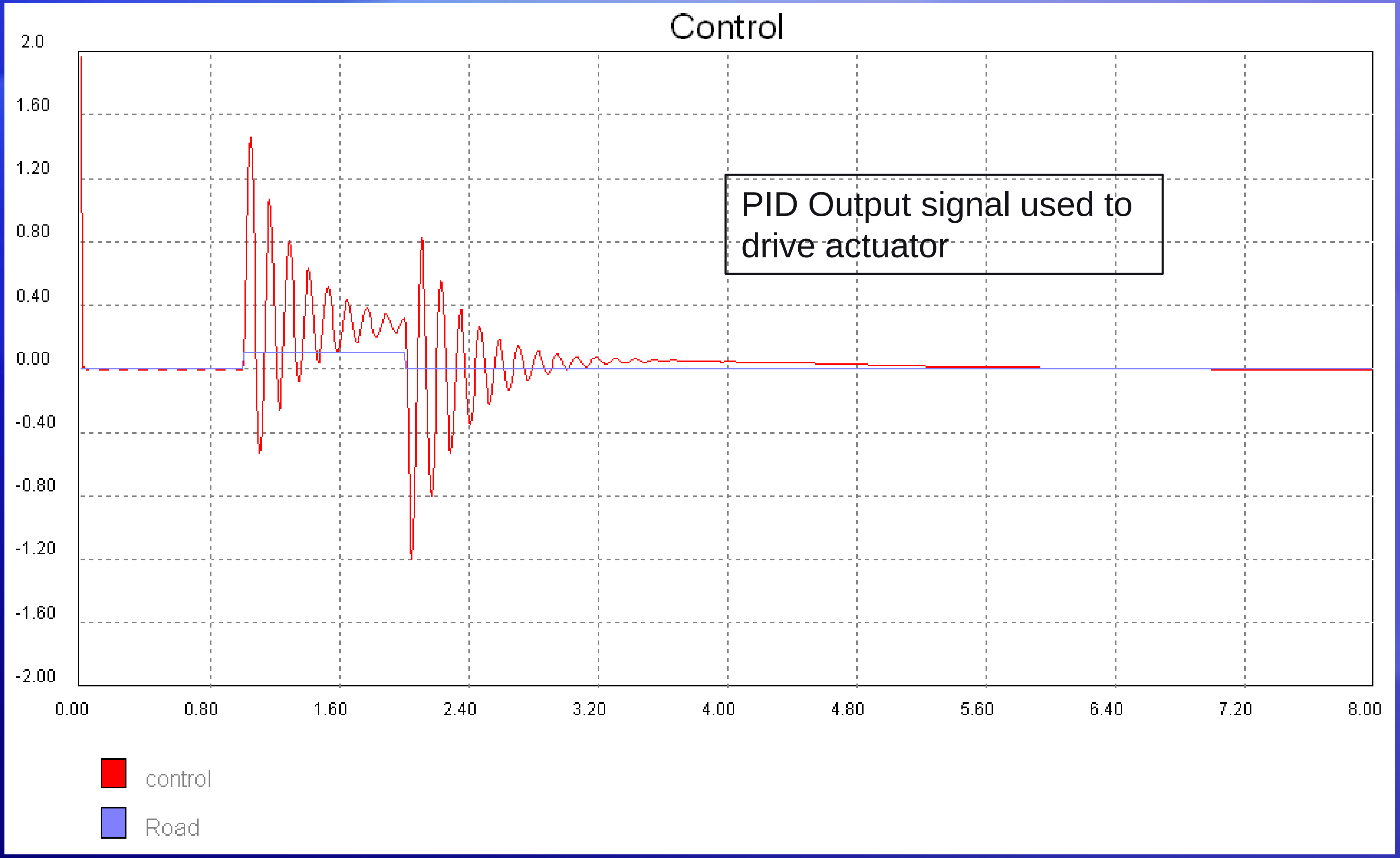
Simulation of Passive and Active Suspension Systems

Comparison: PID Control vs. Passive



Simulation of Passive and Active Suspension Systems

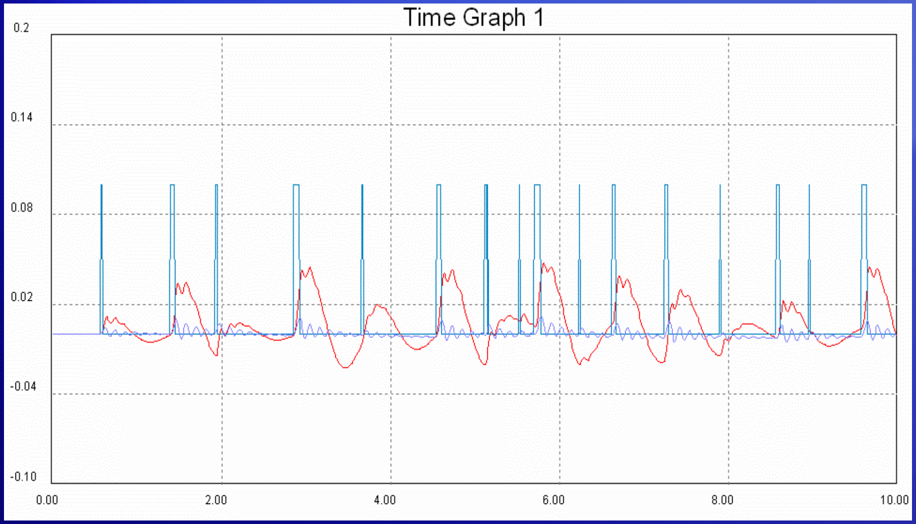
Comparison: PID Control vs. Passive

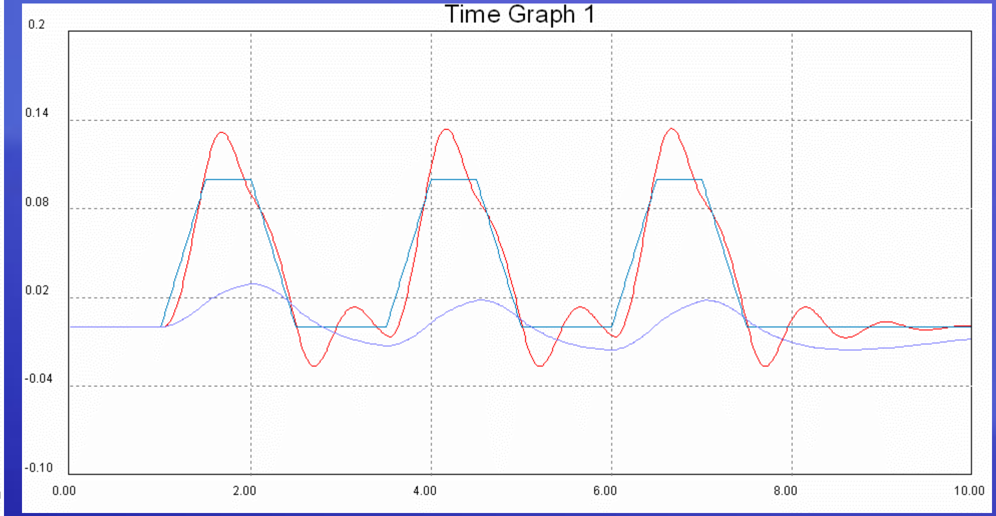


Comparison: PID Control vs. Passive

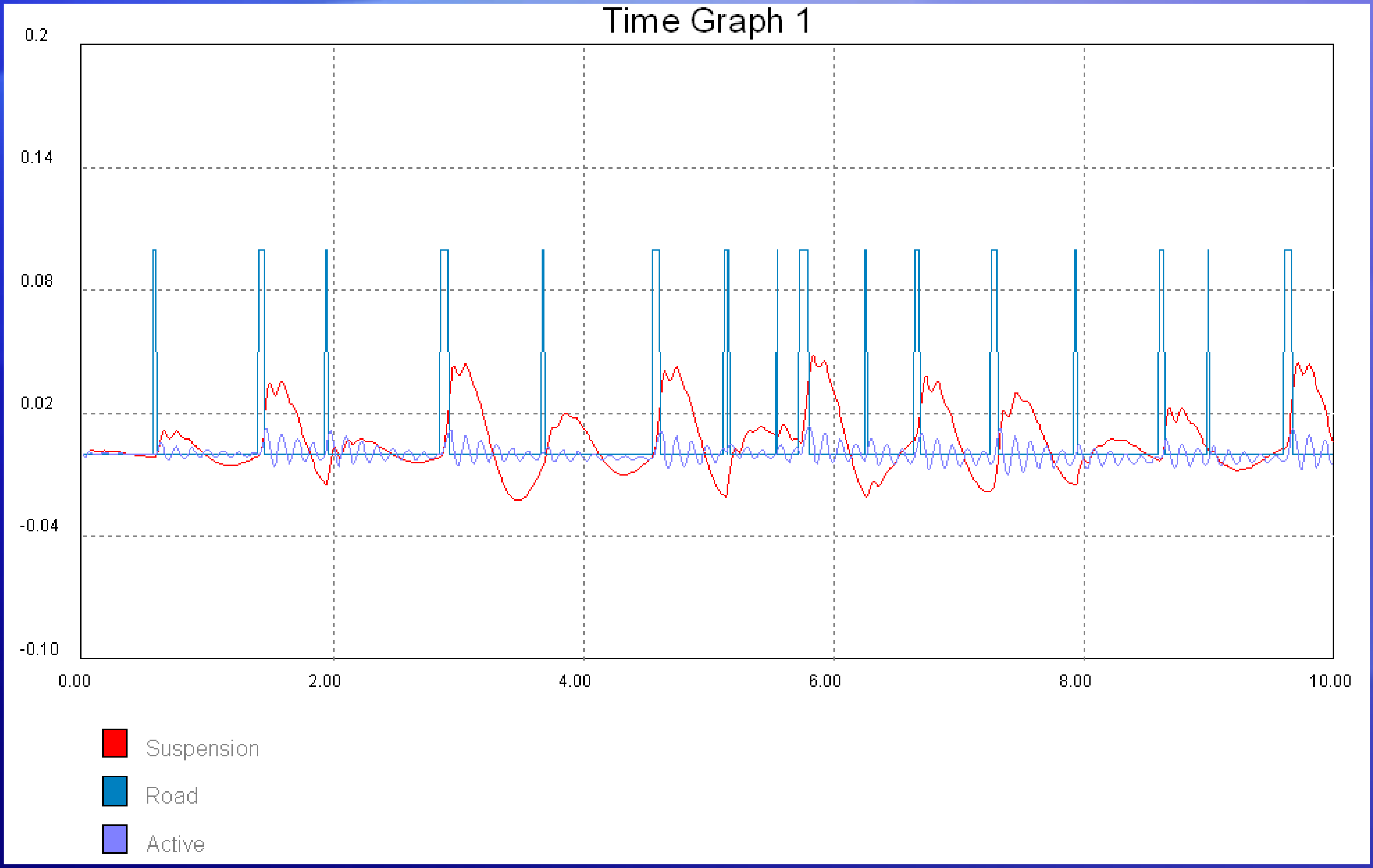
Demo 6 & 7

Great improvement with random and engineered bumps.



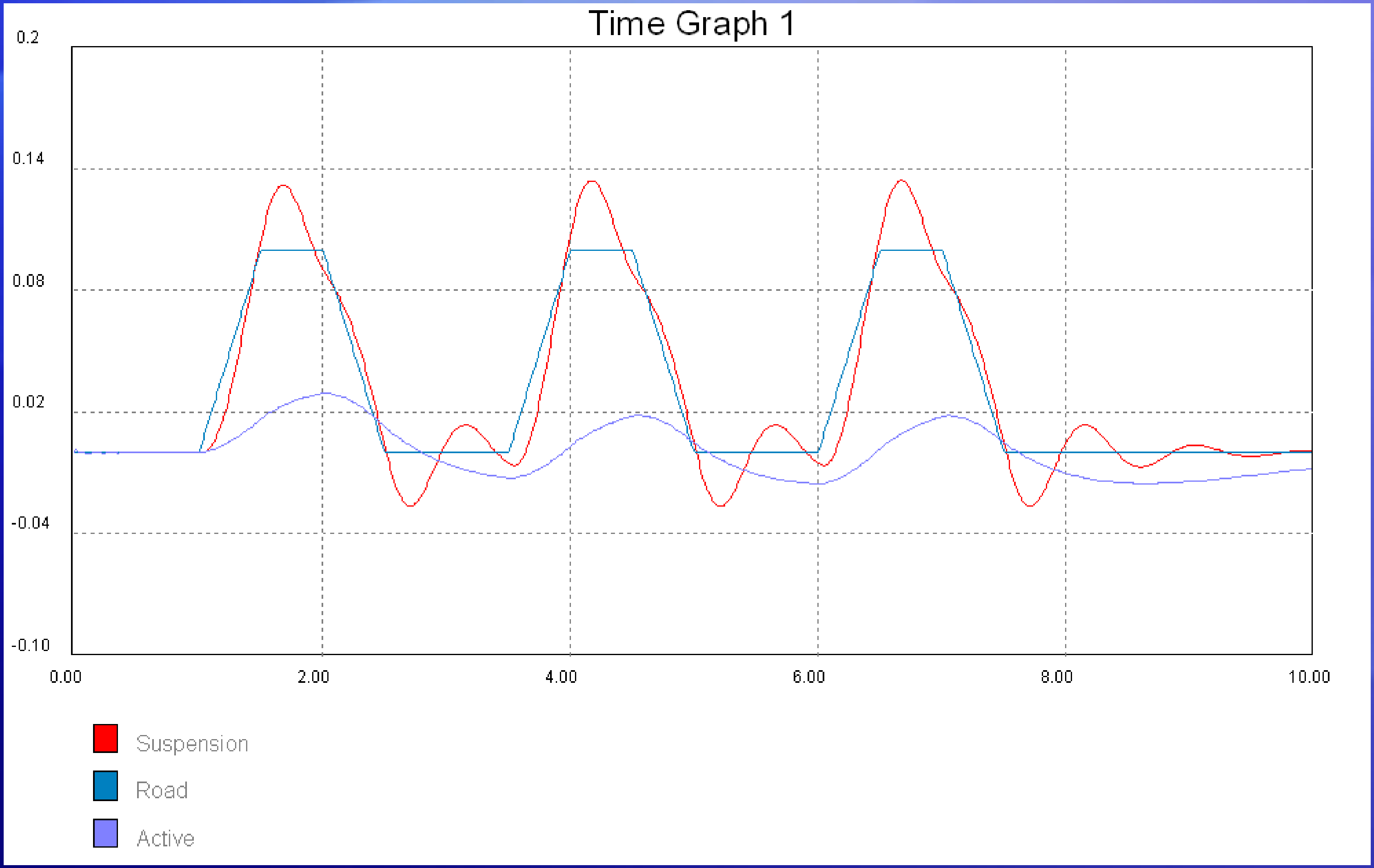


Comparison: PID Control vs. Passive



Simulation of Passive and Active Suspension Systems

Comparison: PID Control vs. Passive



Simulation of Passive and Active Suspension Systems

Limitations

- System assumes very small tyre diameter (tends to a point). Larger tyre diameters have a smoothing effect on road profile, however pre-processing would be required to compute the effective road profile.
- Temperature effect on damper is ignored
- Frequency analysis of the signals was not carried out. Humans are more sensitive to vertical displacements in the region of 4-8Hz[Kru] . Analysis of frequency response will affect selection of PD parameters so that signals in this range are attenuated more.

- Simulation handles only one wheel. Ideally all four wheels and their effect on each other are simulated.

- CSS Frequency analysis.

References

[Lin97] Lin, J., and Kanellakopoulos, I., "Road Adaptive Nonlinear Design of Active Suspensions," *Proceedings of the American Control Conference*, (1997), pp. 714-718. http://citeseer.ist.psu.edu/cache/papers/cs/1806/http:zSzzSzansl.ee.ucla.eduzSzancgzSzaszSzacc97zSzlkacc97.pdf/road-adaptive-nonlinear-design.pdf

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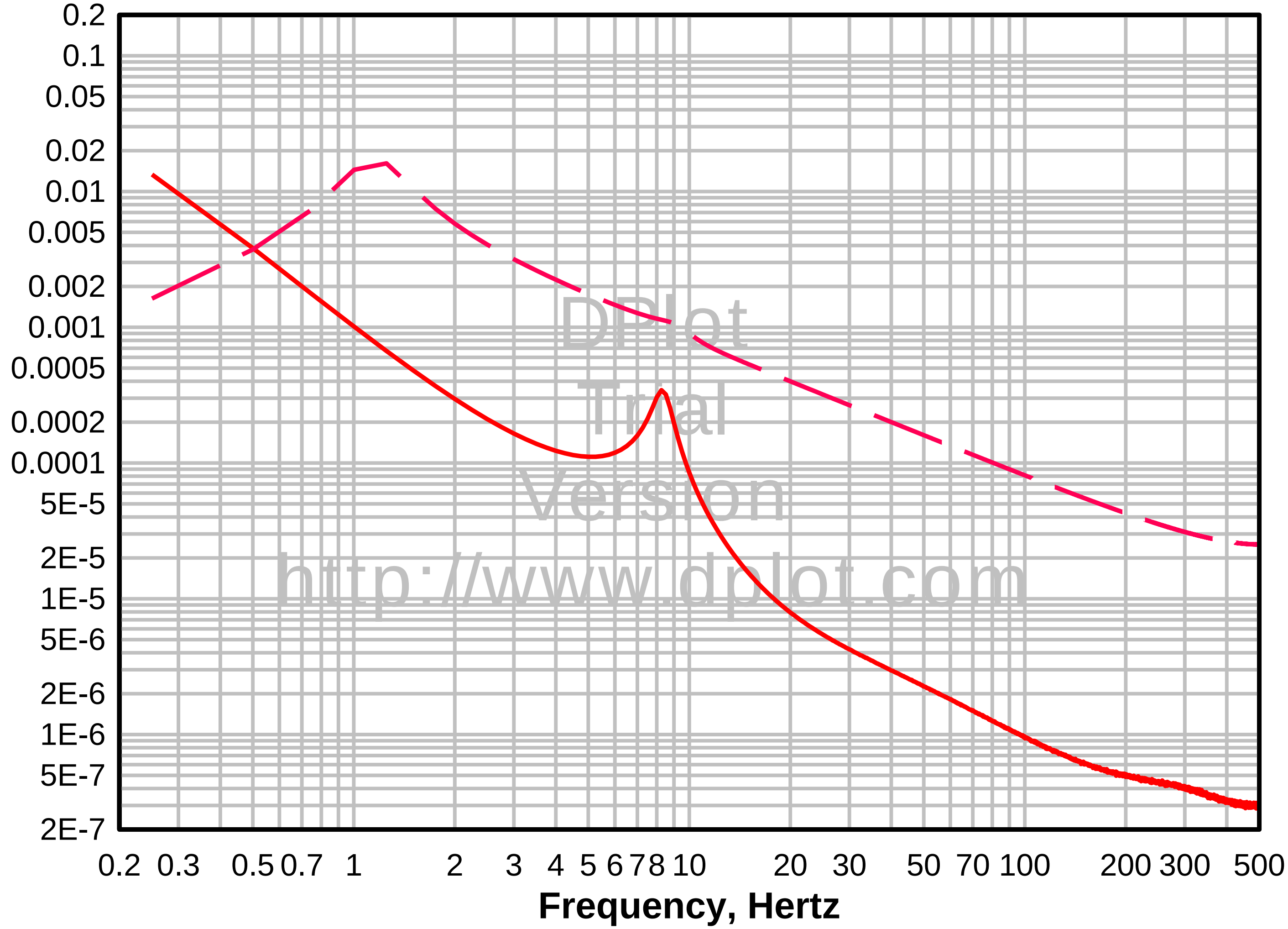
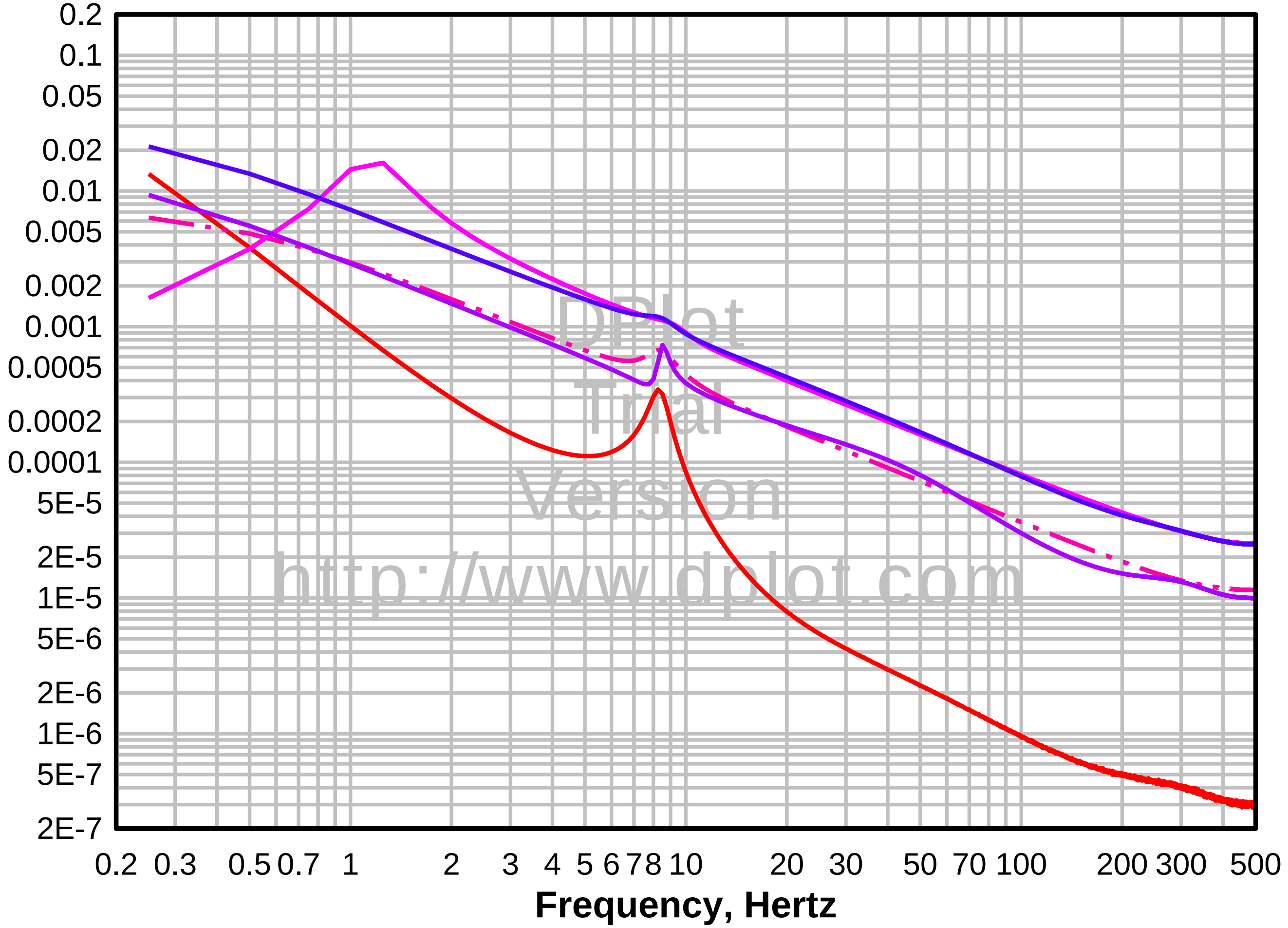
Addendum : Frequency Response

- A trial version of dPlot (www.dplot.com) was used to examine the frequency response characteristics of the systems.
- dPlot imports data directly from Microsoft Excel
- A 'Write to file' Block was used to record simulation data in a text file. File format is [time],[value]
- Data is then exported to dPlot and FFT analysis is executed on the response
- Unit Step Response was analyzed

Frequency Response: Unit Step Response



Frequency Response: Unit Step Response with different PID parameters



Thank you